JEE MAIN Sept. 2020

QUESTION PAPER WITH SOLUTION

MATHEMATICS _ 3 Sep. _ SHIFT - 1

MOTION™
H.O. : 394, Rajeev Gandhi Nagar, Kota
www.motion.ac.in | ✉: info@motion.ac.in

Q.1 The value of $(2.{}^1P_0-3.{}^2P_1+4.{}^3P_2-.....$ up to 51^{th} term$) +(1!-2!+3!-......$ up to 51^{th} term$)$ is equal to:

$(2.{}^1P_0-3.{}^2P_1+4.{}^3P_2-.....$ 51वें पद तक$) +(1!-2!+3!-......$ 51वें पद तक$)$ का मान बराबर है :

(1) 1-51(51)! (2) 1+(52)! (3) 1 (4) 1+ (51)!

Sol. **2**

$2.\ {}^1P_0 = |\underline{2}$

$3.\ {}^2P_1 = |\underline{3}$

$4.\ {}^3P_2 = |\underline{4}$

$(|\underline{2} - |\underline{3} + |\underline{4} - |\underline{5} + ...|\underline{52}) + (|\underline{1} - |\underline{2} + |\underline{3} - |\underline{4} ...... + |\underline{51})$

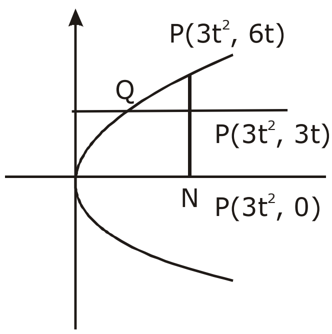
$= |\underline{52} + 1$

Q.2 Let P be a point on the parabola, $y^2=12x$ and N be the foot of the perpendicular drawn from P on the axis of the parabola. A line is now drawn through the mid-point M of PN, parallel to its axis which meets the parabola at Q. If the y-intercept of the line NQ is $\frac{4}{3}$, then:

माना P परवलय, $y^2=12x$ पर एक बिन्दु है और P से परवलय के अक्ष पर डाले गए लम्ब का पाद N है। अब PN के मध्य–बिन्दु M से एक सरल रेखा परवलय के अक्ष के समान्तर खींची जाती है जो परवलय को बिन्दु Q पर मिलती है। यदि रेखा NQ का y-अंतखंड $\frac{4}{3}$ है, तो :

(1) PN=4 (2) MQ=$\frac{1}{3}$ (3) PN=3 (4) MQ=$\frac{1}{4}$

Sol. **4**



Q (h, 3t) lie on
Parabola
$9t^2 = 12h$

$h = \frac{3t^2}{4}$

$Q = \left(\frac{3t^2}{4}, 3t\right)$

Equation of NQ

$y = \frac{3t}{\left(\frac{3t^2}{4} - 3t^2\right)} \quad (x - 3t^2)$

$y = \frac{-4t}{3t^2}\left(x - 3t^2\right)$

put x = 0

$y = \frac{-4}{3t}\left(-3t^2\right) = 4t$

$4t = \frac{4}{3} \Rightarrow t = \frac{1}{3}$

$PN = 6t = 6.\frac{1}{3} = 2$

$M = \left[\frac{1}{3}, 1\right], Q\left[\frac{1}{12}, 1\right]$

$MQ = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$

Q.3 If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$, then B+C is equal to:

यदि $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$ है, तो B+C बराबर है :

(1) 1 (2)-1 (3) -3 (4) 9

Sol. **3**

$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3+Bx^2+Cx+D$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 1 & -x+1 & x-5 \end{vmatrix}$$

$R_3 \to R_3 - R_2$

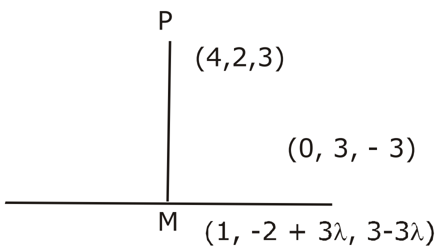
$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & -x+2 & -2x+3 \\ 0 & -1 & 3x-8 \end{vmatrix} = Ax^3+Bx^2+Cx+D$$

$\Rightarrow -1[(3 - 2x)(x - 2) - (3x - 4)] + (3x - 8)[(x - 2)(-x + 2) -(2x - 3)] = Ax^3+Bx^2+Cx+D$
$\Rightarrow 3x - 2x^2 - 6 + 4x - 3x + 4 + (3x - 8)[-x^2 + 4x - 4 - 2x + 3] = Ax^3+Bx^2+Cx+D$
A = -3, B = 12, C = -15
B + C = −3

Q.4 The foot of the perpendicular drawn form the point (4,2,3) to the line joining the points (1,-2,3) and (1,1,0) lies on the plane:
बिन्दुओं (1,-2,3) और (1,1,0) से होकर जाने वाली सरल रेखा पर बिन्दु (4,2,3) से डाले गए लम्ब का पाद जिस समतल पर है है, वह है :
(1) x-y-2z=1 (2) x-2y+z=1 (3)2x+y-z=1 (4) x+2y-z=1

Sol. **3**

$\vec{r} = (1,-2,3) + \lambda(0,3,-3)$



$\overrightarrow{pm} \perp \vec{b}$

$\overrightarrow{pm} . \vec{b} = 0$

$(-3, 3\lambda - 4, -3\lambda) . (0, 3, -3) = 0$

$\Rightarrow 0 + 9\lambda - 12 + 9\lambda = 0 \Rightarrow \lambda = \frac{12}{18} = \frac{2}{3}$

m = (1, 0, 1) are on 2x + y - z = 1

Q.5 If $y^2+\log_e(\cos^2x)=y,\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then

यदि $y^2+\log_e(\cos^2x)=y\ \ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ है, तब

(1) $|y'(0)|+|y''(0)|=1$ (2) $y''(0)=0$
(3) $|y'(0)|+|y''(0)|=3$ (4) $|y''(0)|=2$

Sol. **4**

$2yy' + 2(-\tan x) = y'$ ....(1)

diff. w.r.t.x

$2yy'' + 2(y')^2 - 2\sec^2 x = y''$ ....(2)

Put x = 0 in given equation we get y = 0, 1

from (1) $x = 0,\ y = 0 \Rightarrow y'(0) = 0$

$x = 0,\ y = 1, \Rightarrow y'(0) = 0$

from (2) $x = 0,\ y = 0,\ y'(0) = 0 \Rightarrow y''(0) = -2$

$x = 0,\ y = 1,\ y'(0) = 0 \Rightarrow y''(0) = 2$

$|y''(0)| = 2$

Q.6 $2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{25}\right)$ is equal to:

$2\pi-\left(\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}+\sin^{-1}\frac{16}{25}\right)$ बराबर है :

(1) $\frac{5\pi}{4}$ (2) $\frac{3\pi}{2}$ (3) $\frac{7\pi}{4}$ (4) $\frac{\pi}{2}$

Sol. **2**

$$2\pi-\left[\tan^{-1}\left(\frac{4}{3}\right)+\tan^{-1}\left(\frac{5}{12}\right)+\tan^{-1}\frac{16}{63}\right]$$

$$2\pi-\tan^{-1}\left(\frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}}\right)-\tan^{-1}\left(\frac{16}{63}\right)$$

$$\Rightarrow 2\pi-\tan^{-1}\left(\frac{48+15}{36-20}\right)-\tan^{-1}\left(\frac{16}{63}\right)$$

$$\Rightarrow 2\pi-\left[\tan^{-1}\left(\frac{63}{16}\right)+\cot^{-1}\left(\frac{63}{16}\right)\right]$$

$$\Rightarrow 2\pi-\frac{\pi}{2}=\frac{3\pi}{2}$$

Q.7 A hyperbola having the transverse axis of length $\sqrt{2}$ has the same foci as that of the ellipse $3x^2+4y^2=12$, then this hyperbola does not pass through which of the following points ?

एक अतिपरवलय जिसके अनुप्रस्थ (transverse) अक्ष की लम्बाई $\sqrt{2}$ है और उसके नाभिकेन्द्र, दीर्घवत्त $3x^2+4y^2=12$ के नाभिकेन्द्रों के बराबर हैं। तो अतिपरवलय निम्न में से किस बिन्दु से होकर नहीं जाता ?

(1) $\left(\sqrt{\frac{3}{2}}, \frac{1}{\sqrt{2}}\right)$ (2) $\left(1, -\frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{1}{\sqrt{2}}, 0\right)$ (4) $\left(-\sqrt{\frac{3}{2}}, 1\right)$

Sol. **1**

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$$b_1^2 = a_1^2\left(1-e_1^2\right)$$

$3 = 4(1 - e_1^2)$

$e_1 = \frac{1}{2}$

focus = $(\pm a_1e_1, 0)$
= $(\pm 1, 0)$

Length of transverse axis $2a_2 = \sqrt{2} \rightarrow a_2 = \frac{1}{\sqrt{2}}$

$a_2e_2 = 1$

$= e_2 = \sqrt{2}$

$b_2^2 = a_2^2 (e_2^2 - 1)$

$b_2^2 = \frac{1}{2}(2-1) = \frac{1}{2}$

equation of Hyperbola

$x^2 - y^2 = \frac{1}{2}$

Q.8 For the frequency distribution:

Variate(x):	x_1	x_2	$x_3....x_{15}$
Frequency(f):	f_1	f_2	$f_3.....f_{15}$

where $0<x_1<x_2<x_3<....<x_{15}=10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be:

बारंबारता बंटन

चर (x):	x_1	x_2	$x_3....x_{15}$
बारंबारता (f):	f_1	f_2	$f_3.....f_{15}$

जहाँ $0<x_1<x_2<x_3<....<x_{15}=10$ तथा $\sum_{i=1}^{15} f_i > 0$ है, का मानक विचलन, निम्न में से कौन–सा नहीं हो सकता ?

(1) 1 (2) 4 (3) 6 (4) 2

Sol. **3**

$$\sigma^2 \le \frac{1}{4}(M - m)^2$$

(M = upper bound of value of any random variable,
m = Lower bound of value of any random variable)

$$\sigma^2 \le \frac{1}{4}(10 - 0)^2$$

$\sigma^2 < 25$

$-5 < \sigma < 5$

$\sigma \neq 6$

Q.9 A die is thrown two times and the sum of the scores appearing on the die is observed to be a multiple of 4. Then the conditional probability that the score 4 has appeared atleast once is:

एक पासा दो बार फेंका जाता है तथा पासों पर आयी संख्याओं का योगफल 4 का एक गुणज है। तो संख्या 4 के कम से कम एक बार आने की सप्रतिबंध प्रायिकता है :

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{8}$ (4) $\frac{1}{9}$

Sol. **4**

Total Possibilities = (1, 3), (3, 1), (2, 2),
(2, 6), (6, 2) (4, 4)
(3, 5), (5, 3) (6, 6)
fav. = 1 = (4, 4)

prob. = $\frac{1}{9}$

Q.10 If the number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ is exactly 33, then the least value of n is:

यदि $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$ के प्रसार में प्रर्णाकीय पदों की संख्या मात्र 33 है, तो n का न्यूनतम मान है।

(1) 128 (2) 248 (3) 256 (4) 264

Sol. **3**

$$T_{r+1} = {}^nC_r \left(3^{\frac{1}{2}}\right)^{n-r} \left(5^{\frac{1}{8}}\right)^r$$

$\frac{n-r}{2} \rightarrow$ n - r = 0, 2, 4, 6, 8, ...

$\frac{r}{8} \rightarrow r = 0, 8, 16, 24.................$

common r = 0, 8, 16, 24.........
no. of integral term = 33.
L = 0 + (33 - 1) × 8 → L = 32 × 8
= 256

Q.11 $\int_{-\pi}^{\pi} |\pi - |x|| \, dx$ is equal to:

$\int_{-\pi}^{\pi} |\pi - |x|| \, dx$ का मान है :

(1) π^2 (2) $\frac{\pi^2}{2}$ (3) $\sqrt{2}\pi^2$ (4) $2\pi^2$

Sol. **1**

$\int_{-\pi}^{\pi} \left|\pi - |x|\right| dx$

even function

$2\int_{0}^{\pi} |\pi - x| dx$

$= 2\int_{0}^{\pi} (\pi - x) dx \Rightarrow 2\left[\pi x - \frac{x^2}{2}\right]_0^{\pi}$

$= 2\left[\frac{\pi^2}{2}\right] = \pi^2$

Q.12 Consider the two sets:
A={m ∈ R : both the roots of $x^2-(m+1)x+m+4=0$ are real} and B=[-3,5).
Which of the following is not true ?
निम्न दो समुच्चयों पर विचार कीजिए :
A={m ∈ R : $x^2-(m+1)x+m+4=0$ के दोनों मूल वास्तविक हैं} तथा B=[-3,5).
निम्न में से कौन सा सत्य नहीं है ?

(1) $A - B = (-\infty, -3) \cup (5, \infty)$ (2) $A \cap B = \{-3\}$

(3) B-A=(-3,5) (4) $A \cup B = R$

Sol. **1**
$D \geq 0$
$(m + 1)^2 - 4(m + 4) \geq 0$
$\Rightarrow m^2 - 2m - 15 \geq 0$
$(m - 5)(m + 3) \geq 0$
$m \in (-\infty, -3] \cup [5, \infty)$

$A = (-\infty, -3] \cup [5, \infty)$
$B = [-3, 5)$
$A - B = (-\infty, -3) \cup [5, \infty)$
$A \cup B = R$

Q.13 The proposition $p \rightarrow \sim(p \wedge \sim q)$ is equivalent to :

साध्य $p \rightarrow \sim(p \wedge \sim q)$ निम्न में से किसके तुल्य है ?

(1) $(\sim p) \vee (\sim q)$ (2) $(\sim p) \wedge q$

(3) q (4) $(\sim p) \vee q$

Sol. **4**

$\sim(p \wedge \sim q) \rightarrow \sim p \vee q$
$p \rightarrow (\sim p \vee q)$
$\Rightarrow \sim p \vee (\sim p \vee q)$
$\Rightarrow \sim p \vee q$

Q.14 The function, $f(x) = (3x-7)x^{2/3}$, $x \in R$ is increasing for all x lying in:
फलन $f(x) = (3x-7)x^{2/3}$, $x \in R$ के वर्धमान होने के लिए सभी x निम्नलिखित में स्थित है ?

(1) $\left(-\infty, -\frac{14}{15}\right) \cup (0, \infty)$ (2) $\left(-\infty, \frac{14}{15}\right)$

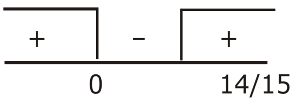
(3) $(-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$ (4) $(-\infty, 0) \cup \left(\frac{3}{7}, \infty\right)$

Sol. **3**

$$f(x) = (3x - 7).\frac{2}{3x^{\frac{1}{3}}} + x^{\frac{2}{3}}.3$$

$$= \frac{6x - 14 + 9x}{3x^{\frac{1}{3}}}$$

$$= \frac{15x - 14}{3x^{\frac{1}{3}}}$$



$f(x) > 0 \uparrow \Rightarrow x \in (-\infty, 0) \cup \left(\frac{14}{15}, \infty\right)$

Q.15 If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is:
यदि एक समांतर श्रेढ़ी का प्रथम पद 3 है तथा इसके प्रथम 25 पदों का योग, इसके अगले 15 पदों के योग के बराबर है, तो इस समांतर श्रेढ़ी का सार्वअंतर है :

(1) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{7}$

Sol. **1**
a = 3

$$\frac{25}{2}[2a+24d]=\frac{15}{2}[2\times(a+25d)+14d]$$

$\Rightarrow$ 50a + 600d = 15 [2a + 50d + 14d]
$\Rightarrow$ 20a + 600d = 960d
$\Rightarrow$ 60 = 360d

$d=\frac{1}{6}$

Q.16 The solution curve of the differential equation, $(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2$, which passes through the point (0,1), is:

अवकल समीकरण $(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2$, का हल वक्र, जो बिन्दु (0,1) से होकर जाता है, है :

(1) $y^2=1+y\log_e\left(\frac{1+e^{-x}}{2}\right)$ (2) $y^2+1=y\left(\log_e\left(\frac{1+e^{-x}}{2}\right)+2\right)$

(3) $y^2+1=y\left(\log_e\left(\frac{1+e^{x}}{2}\right)+2\right)$ (4) $y^2=1+y\left(\log_e\left(\frac{1+e^{x}}{2}\right)\right)$

Sol. **4**

$$\int\left(\frac{1+y^2}{y^2}\right)dy=\int\left(\frac{1}{1+e^{-x}}\right)dx$$

$$\int\frac{1}{y^2}dy+\int dy=\int\left(\frac{e^x}{e^x+1}\right)dx$$

$$\Rightarrow \frac{-1}{y}+y=\ln\left|e^x+1\right|+c$$

x = 0, y = 1
$\Rightarrow$ -1 + 1 = ln2 + c $\Rightarrow$ c = -ln2

$$\Rightarrow \frac{-1}{y}+y=\ln\left|e^x+1\right|-\ln 2$$

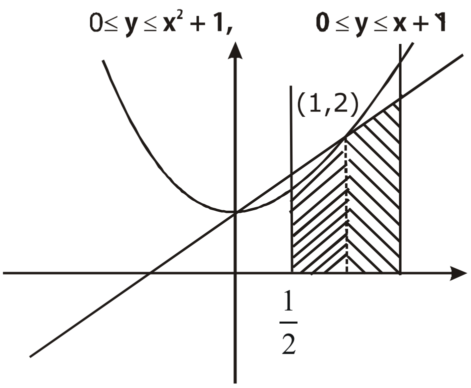
$$\Rightarrow \; y^2 = 1 + y\left[\ln\left(\frac{e^x + 1}{2}\right)\right]$$

Q.17 The area (in sq. units) of the region $\left\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\right\}$ is

क्षेत्र $\left\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\right\}$ का क्षेत्रफल (वर्ग इकाइयों में) है :

(1) $\frac{23}{16}$ (2) $\frac{79}{16}$ (3) $\frac{23}{6}$ (4) $\frac{79}{24}$

Sol. **4**



$$A = \int_{\frac{1}{2}}^{1}(x^2 + 1)dx + \int_{1}^{2}(x + 1)\,dx$$

$$\left(\frac{x^3}{3} + x\right)_{\frac{1}{2}}^{1} + \left(\frac{x^2}{2} + x\right)_{1}^{2}$$

$$= \left(\frac{1}{3} + 1\right) - \left(\frac{1}{24} + \frac{1}{2}\right) + \left((2 + 2) - \left(\frac{3}{2}\right)\right)$$

$$= \left(\frac{4}{3} - \frac{13}{24}\right) + \left(\frac{5}{2}\right)$$

$$= \left(\frac{32 - 13}{24}\right) + \left(\frac{5}{2}\right) = \frac{19 + 60}{24} = \frac{79}{24}$$

Q.18 If α and β are the roots of the equation $x^2+px+2=0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2+2qx+1=0$, then $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ is equal to :

यदि α तथा β समीकरण $x^2+px+2=0$ के मूल है तथा $\frac{1}{\alpha}$ एवं $\frac{1}{\beta}$, समीकरण $2x^2+2qx+1=0$ के मूल हैं, तो $\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$ बराबर है :

(1) $\frac{9}{4}(9+p^2)$ (2) $\frac{9}{4}(9+q^2)$ (3) $\frac{9}{4}(9-p^2)$ (4) $\frac{9}{4}(9-q^2)$

Sol. **3**

$\alpha + \beta = -p, \alpha\beta = 2$

$\frac{1}{\alpha}+\frac{1}{\beta} = -q, \frac{1}{\alpha\beta} = \frac{1}{2}$

$\frac{\alpha+\beta}{\alpha\beta} = -q \Rightarrow \frac{-p}{2} = -q$

$\Rightarrow p = 2q$

$\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2$

$= 2 + \frac{1}{2} + 2 = \frac{9}{2}$

$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right) = \alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}$

$= 2 + \frac{1}{2} - \left[\frac{\alpha^2+\beta^2}{\alpha\beta}\right]$

$= \frac{5}{2} - \left[\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}\right]$

$= \frac{5}{2} - \left[\frac{p^2-4}{2}\right]$

$= \frac{9-p^2}{2}$

$\left(\alpha-\frac{1}{\alpha}\right)\left(\beta-\frac{1}{\beta}\right)\left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)=\left(\frac{9-p^2}{2}\right)\left(\frac{9}{2}\right)$

$= \frac{9}{4}\left(9-p^2\right)$

Q.19 The lines $\vec{r}=\left(\hat{i}-\hat{j}\right)+l\left(2\hat{i}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}\right)+m\left(\hat{i}+\hat{j}-\hat{k}\right)$

(1) do not intersect for any values of l and m
(2) intersect when l=1 and m=2
(3) intersect when l=2 and m=$\frac{1}{2}$
(4) intersect for all values of l and m

सरल रेखाएँ $\vec{r}=\left(\hat{i}-\hat{j}\right)+l\left(2\hat{i}+\hat{k}\right)$ तथा $\vec{r}=\left(2\hat{i}-\hat{j}\right)+m\left(\hat{i}+\hat{j}-\hat{k}\right)$

(1) l तथा m के किसी भी मान के लिए नही काटती है।
(2) काटती है, जब l =1 तथा m=2
(3) काटती है, जब l=2 तथा m=$\frac{1}{2}$
(4) l तथा m के सभी मानो के लिए काटती है।

Sol. **1**

$\frac{2}{1}\neq\frac{0}{1}\neq\frac{1}{-1}$ → lines are intersecting

$\vec{r}=\left(1+2l\right)\hat{i}-\hat{j}+l\hat{k}$ ...(1)

$\vec{r}=\left(2+m\right)\hat{i}+\left(m-1\right)\hat{j}-m\hat{k}$ ....(2)

compare coff. of $\hat{i},\hat{j},\hat{k}$

$1+2l=2+m$ $\quad \left| \begin{matrix} -1=m-1 \\ m=0 \end{matrix} \right|$ $\quad l=0$

Lines do not intersect

Q.20 Let [t] denote the greatest integer $\le$t. if for some $\lambda \in R - \{0, 1\}$

$\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x + [x]}\right| = L$, then L is equal to:

माना [t] महत्तम पूर्णांक $\le$t को दर्शाता है। यदि किसी $\lambda \in R - \{0, 1\}$

$\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x + [x]}\right| = L$ है, तो L का मान है :

(1) 0 (2) 2 (3) $\frac{1}{2}$ (4) 1

Sol. **2**

$$\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda - x+[x]}\right| = L$$

$$\lim_{h\to 0}\left|\frac{1-h+h}{\lambda - h + [h]}\right|$$

$$\lim_{h\to 0}\left|\frac{1}{\lambda - h + 0}\right| = \left|\frac{1}{\lambda}\right| \quad [x] = 0$$

$$\lim_{h\to 0}\left|\frac{1+h+h}{\lambda + h + [-h]}\right|$$

$$= \left|\frac{1}{\lambda - 1}\right| \quad [-h] = -1$$

$\because$ $|\lambda| = |\lambda - 1|$

$\lambda^2 = \lambda^2 - 2\lambda + 1 \Rightarrow \lambda = \frac{1}{2}$

L = 2

Q.21 If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$, then the vlaue of k is ....

यदि $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\} = 2^{-k}$, तो k का मान है ....

Sol. **8**

$$\lim_{x\to 0}\frac{\left(1-\cos\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)^2}\frac{\left(1-\cos\frac{x^2}{4}\right)}{\left(\frac{x^2}{4}\right)^2}\cdot\frac{\left(\frac{x^2}{2}\right)\cdot\left(\frac{x^2}{4}\right)^2}{x^8}$$

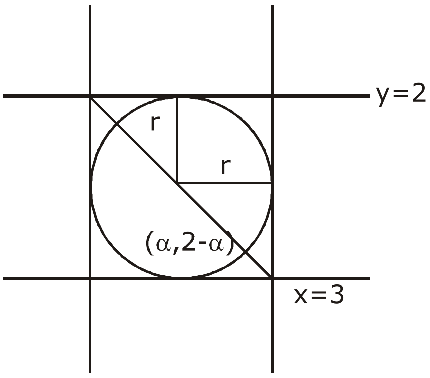
$$\lim_{x\to 0}\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{16}\Rightarrow\frac{1}{256}=2^{-k}$$

$2^{-8} = 2^{-k} \Rightarrow k = 8$

Q.22 The diameter of the circle, whose centre lies on the line x + y = 2 in the first quadrant and which touches both the lines x=3 and y=2, is ....

वृत्त, जिसका केन्द्र प्रथम चतुर्थांश में रेखा x + y = 2 पर हैं तथा जो दोनों रेखाओं x=3 तथा y=2 को स्पर्श करता है, का व्यास है .... ।

Sol. **2**



p = r

for y = 2

$$r=\left|\frac{2-\alpha-2}{1}\right|=|\alpha|$$

for x = 3

$$r=\left|\frac{\alpha-3}{1}\right|=|\alpha-3|$$

$|\alpha| = |\alpha - 3|$

$\Rightarrow \alpha^2 + \alpha^2 - 6\alpha + 9 \Rightarrow \alpha = \frac{3}{2}$

$2\alpha = 3 = 2r$

Q.23 The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+..........\text{to }\infty\right)}$ is equal to..............

$(0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+..........\text{to }\infty\right)}$ का मान है.............. ।

Sol. **4**

$$\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+........\infty=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{1}{2}$$

$$\log_{2.5}\left(\frac{1}{2}\right) \Rightarrow \log_{\frac{5}{2}}\frac{1}{2}$$

$$.16=\frac{16}{100}=\frac{4}{25}=\left(\frac{2}{5}\right)^2$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2\log_{\frac{5}{2}}\frac{1}{2}}=\left(\frac{5}{2}\right)^{-2\log_{\frac{5}{2}}\frac{1}{2}}$$

$$\Rightarrow \left(\frac{5}{2}\right)^{\log_{\frac{5}{2}}\left(\frac{1}{2}\right)^{-2}}$$

$= 4$

Q.24 Let $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ and $A^4=[a_{ij}]$. If $a_{11}=109$, then a_{22} is equal to ....

माना $A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}, x \in R$ तथा $A^4=[a_{ij}]$ है। यदि $a_{11}=109$ है, तो a_{22} बराबर है .... ।

Sol. **10**

$$A=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2=\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^3+x+x & x^2+1 \\ x^2+1 & x \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^3+2x & x^2+1 \\ x^2+1 & x \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x^4+2x^2+x^2+1 & x^3+2x \\ x^3+x+x & x^2+1 \end{bmatrix}$$

$a_{11} \Rightarrow x^4 + 3x^2 + 1 = 109$
$x^4 + 3x^2 - 108 = 0$
$\Rightarrow (x^2 + 12)(x^2 - 9) = 0$
$x = \pm 3$
$a_{11} = x^2 + 1 = 10$

Q.25 If $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, $(m, n \in N)$ then the greatest common divisor of the least values of m and n is ....

यदि $\left(\frac{1+i}{1-i}\right)^{\frac{m}{2}} = \left(\frac{1+i}{i-1}\right)^{\frac{n}{3}} = 1$, $(m, n \in N)$ तो m तथा n के न्यूनतम मानों का महत्तम उभयनिष्ठ भाजक है ....।

Sol. **4**

$$\left[\frac{(1+i)(1+i)}{(1+i)(1-i)}\right]^{\frac{m}{2}} = \left[\left(\frac{1+i}{-1+i}\right)\left(\frac{-1-i}{-1-i}\right)\right]^{\frac{n}{3}} = 1$$

$$= \left(\frac{2i}{2}\right)^{\frac{m}{2}} = 1 \quad \Bigg| \quad \left(\frac{-1-i-i+1}{1+1}\right)^{\frac{n}{3}} = 1$$

m = 8

$(-i)^{n/3} = 1$
n = 12

greatest common divisor of m & n is 4